

## Stress-strain state of a drill string in well sections with “dog-legs”

R.V. Rachkevych \*

Ivano-Frankivsk National Technical University of Oil and Gas;  
15, Karpatska Str., Ivano-Frankivsk, 76019, Ukraine

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## Abstract

During the research a drill string was simulated by a flexible beam. The flexible beam has a 3D deformation under external loadings. The system of vector differential equations of equilibrium is used for analysis of the stress-strain state. The mathematical model takes into consideration a body weight and stiffness of a drill string, axial force, torsional moment and reactions of a hole wall.

The deformation of a drill string elastic line with external diameter of 127 mm in a well “dog-leg” was analyzed in real operational conditions using the developed method. The author obtained the function of a drill string elastic line in 3D coordinate system. There were drawn the diagrams of normal stresses of bending around basic axes of inertia of a drill string cross-section. It is also noticed that the “dog-leg” increases these stresses twice.

Keywords: a drill string, a flexible beam, a stress-strain state, “dog-leg” of a well, three-dimensional elastic line.

The practice of drilling in Ukraine and abroad shows that many wells have places where inclination and horizontal angles change; specialists call them “abrupt bend” or “dog-legs” [1–5]. The tendency and intensity of angles change at these places do not represent intermediate tendency and intensity of change of well angles. Therefore “dog-legs” can be at rectilinear and curvilinear places of wells.

A. Lubinskyi and H. Woods were the first to research this effect [6]. The scientists described the reasons of “dog-legs” formation. A. Lubinskyi and H. Woods specified the basic reasons. They are an abrupt change of bit load and characteristics of geological materials, esp. the change of inclination angle of adjacent stratum (Fig. 1).

The authors of a research paper [7] write that the basic reason of “dog-legs” forming is also rotation of stem assembly around its rotation axis.

But “dog-legs” provoke additional deformation of a drill string, indentation of box-and-pin joints and pipes body to the side of a hole [6]. This effect results in increase of normal stresses of arch in cross sections of drill pipes; fast deterioration of box-and-pin joints and pipes body; formation of a ditch on side of a hole; deterioration of boring casing, which leads to deformation in operation period [6]. Besides, “dog-legs” provoke complications at extraction of oil and gas [6].

The authors of the research paper [6] recommend to avoid the presence of “dog-legs” with the maximal

degree of crookedness 1.65 grad/10 m in vertical and directionally drilled wells for the “abrupt bend” control. But A. Lubinskyi and H. Woods point out that admissible degree of crookedness of “abrupt bend” depends on the distance between “dog-legs” and the hole bottom, diameters of a well, drill pipes, case pipes, oil-well tubing and rods. The influence of these factors on the maximal degree of crookedness will be clearer after additional theoretical researches and accumulation of industrial data.

Therefore, the aim of the work is to define the stress-strain state of a drill, which is in a “dog-leg”, when inclination and horizontal angles change, using analytical method.

We study a 3D axis of a wellbore, which is set discretely during the drillhole survey. Usually we get  $m$  points in this case. For each point, there are set the depth  $a_i$ , an inclination angle  $\alpha_i$  and a horizontal angle  $\gamma_i$  ( $i = 1 \dots m$ ). After the processing of final results we get the absolute depth –  $h_{1i}$ , horizontal displacement along “South – North” axes –  $h_{2i}$  and horizontal displacement along “West – East” axes –  $h_{3i}$ .

We also define coordinates of the points for accounting the movement restrictions between drill string axis and well axis. We call these points as the “top side” and the “lower side” of the hole.

Let’s assume that  $Y$  is any point of a drillhole survey (Fig. 2). We draw a tangent to the well axis in this point. The angle between the tangential and vertical lines is marked as  $\alpha$ . The angle between the plan of the tangent and  $X_2$  axis is marked as  $\gamma$ .

Consequently, the angles between the vertical and projection of the tangential line on plans  $X_1O X_2$  and  $X_1O X_3$  and are equal to

$$\alpha_{12} = \arctan(\tan \alpha \cos \gamma),$$

$$\alpha_{13} = \arctan(\tan \alpha \sin \gamma).$$

\* Corresponding author:  
ruslantkachuk79@gmail.com

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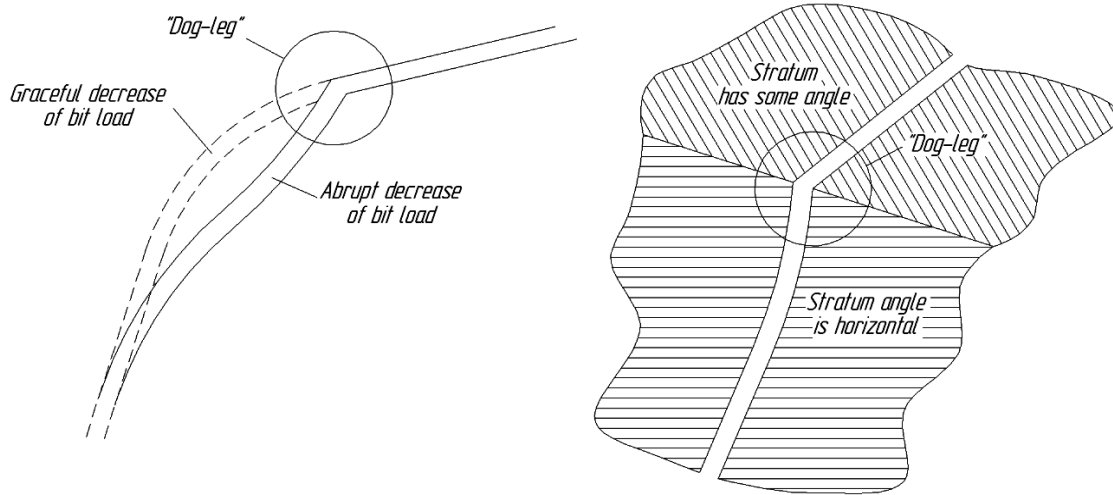


Figure 1 – “Dog-leg” of a well

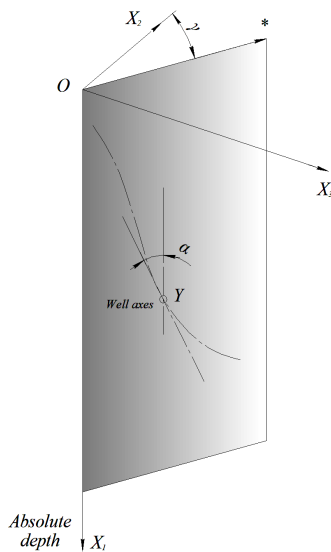


Figure 2 – An inclination angle  $\alpha_i$  and a horizontal angle  $\gamma_i$  for point Y

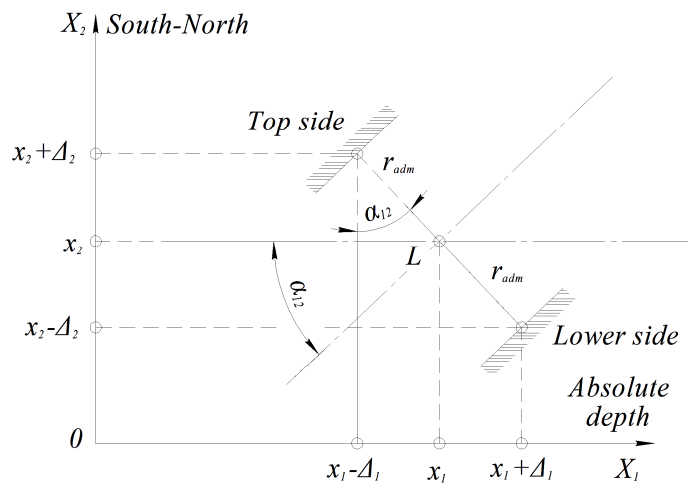


Figure 3 – Geometrical plotting for defining the coordinates of points at the “top side” and the “lower side” of a hole

We also define non-dimensional coordinates of points at the “top side” and the “lower side” of a hole in a plane  $X_1OX_2$  (Fig. 3).

The result is the following:

$$\Delta_1 = r_{adm} \sin \alpha_{12},$$

$$\Delta_2 = r_{adm} \cos \alpha_{12}.$$

The variable  $r_{adm}$  is a non-dimensional value of a radial clearance and it is defined by the formula

$$r_{adm} = \frac{D - d}{2l_b},$$

where  $D$  and  $d$  are external and internal diameters of a drill string (availability of box-and-pin joint isn't taken into consideration in this formula and the following calculations, that is why the value of  $d$  equals the external diameter of a drill pipe body).

The length of an elastic beam  $l_b$  for modeling a hole interval with a “dog-leg” is

$$l_b = {}_a l_{k2} - {}_a l_{k1},$$

where  ${}_a l_{k1}$ ,  ${}_a l_{k2}$ ,  $k1$ ,  $k2$  are the depths and the order numbers of drillhole survey points, which correspond to the beginning and the ending of a hole interval under investigation.

Therefore, we can write down the coordinates of the points

$$[x_1 - \Delta_1, x_2 + \Delta_2] - \text{top side},$$

$$[x_1 + \Delta_1, x_2 - \Delta_2] - \text{lower side}.$$

Here

$$x_1 = (h_1 - h_{1k1}) / l_b,$$

$$x_2 = (h_2 - h_{2k1}) / l_b,$$

where  $h_{1k1}$ ,  $h_{2k1}$  are the absolute depth and horizontal displacement along “South – North” axes for the point at the beginning of a hole interval under investigation.

We can apply an analogical idea for the plane  $X_1OX_3$ . As a result

$$\Delta_3 = r_{adm} \cos \alpha_{13}.$$

Coordinates of the point of “top side” and “lower side” are

$$[x_1 - \Delta_1, x_3 + \Delta_3] - \text{top side},$$

$$[x_1 + \Delta_1, x_3 - \Delta_3] - \text{lower side}.$$

As in the previous case

$$x_3 = (h_3 - h_{3kl})/l_b,$$

where  $h_{3kl}$  is a horizontal displacement along “West – East” axes for the point at the beginning of a hole interval, which is examined.

Now we define coordinates of points of hole sides for the plane  $X_2OX_3$ . By analogy to Fig. 3 we have

$$\Delta_{23} = r_{adm} \sin \gamma,$$

$$\Delta_{32} = r_{adm} \cos \gamma.$$

Therefore,

$$[x_3 - \Delta_{32}, x_2 + \Delta_{23}] - \text{right side},$$

$$[x_3 + \Delta_{32}, x_2 - \Delta_{23}] - \text{left side}.$$

Elastic line of a drill string is a set of points with coordinates  $[\varepsilon + u_{x1}(\varepsilon), u_{x2}(\varepsilon), u_{x3}(\varepsilon)]$ ,  $\varepsilon = 0..1$ , because the stress-strain state is analyzed by the numerical method. As a result we can write the conditions that the distance between a drill column and a hole side in this point is within the radial clearance (see Table 1).

**Table 1 – Conditions for defining a drill column position in a well bore**

| Conditions   | Plane     |
|--|-----------|
| $x_1 - \Delta_1 \leq \varepsilon + u_{x1}(\varepsilon) \leq x_1 + \Delta_1$<br>$x_2 - \Delta_2 \leq u_{x2}(\varepsilon) \leq x_2 + \Delta_2$ | $X_1OX_2$ |
| $x_1 - \Delta_1 \leq \varepsilon + u_{x1}(\varepsilon) \leq x_1 + \Delta_1$<br>$x_3 - \Delta_3 \leq u_{x3}(\varepsilon) \leq x_3 + \Delta_3$ | $X_1OX_3$ |
| $x_2 - \Delta_{23} \leq u_{x2}(\varepsilon) \leq x_2 + \Delta_{23}$<br>$x_3 - \Delta_{32} \leq u_{x3}(\varepsilon) \leq x_3 + \Delta_{32}$   | $X_2OX_3$ |

If conditions (see Table 1) are true, the distance between well axis and drill string axis is in the range of a radial clearance.

For analysis of the stress-strain state of a drill string we can simulate it by a flexible beam [8], for which the system of vector equations of equilibrium is the following [9]

$$\frac{dQ}{d\varepsilon} + \chi \times Q + P = 0, \quad (1)$$

$$\frac{dM}{d\varepsilon} + \chi \times M + \bar{e}_1 \times Q + T = 0, \quad (2)$$

$$M = A(\chi - \chi_0^{(1)}), \quad (3)$$

$$L \frac{d\vartheta}{d\varepsilon} + L_2 \chi_0^{(1)} - A^{-1} M = 0, \quad (4)$$

$$\frac{du}{d\varepsilon} + \chi \times u + (l_{11} - 1)e_1 + l_{21}e_2 + l_{31}e_3 = 0, \quad (5)$$

where  $Q$  and  $M$  are vectors of internal forces and moments;  $\varepsilon$  is a non-dimensional coordinate;  $P, T$  are vectors of external forces and moments;  $\chi, \chi^{(1)}$  are vectors of current and initial curvature of beam;  $e_1, e_2, e_3$  are unit vectors of moving coordinate system (it moves the system of coordinate with axes, the direction of which coincides with the direction of basic beam axes of inertia);  $A$  is a matrix of the beam stiffness;  $L, L_2$  are transformation matrixes between vector bases;  $\vartheta$  is a vector of rotation angle of moving coordinate

system relatively to its initial position;  $u$  is a displacement vector;  $l_{11}, l_{21}, l_{31}$  are elements of the matrix  $L$ .

The vectors  $P$  and  $T$  are defined by the formulas

$$P = q + \sum_{i=1}^n P^{(i)} \delta(\varepsilon - \varepsilon_i),$$

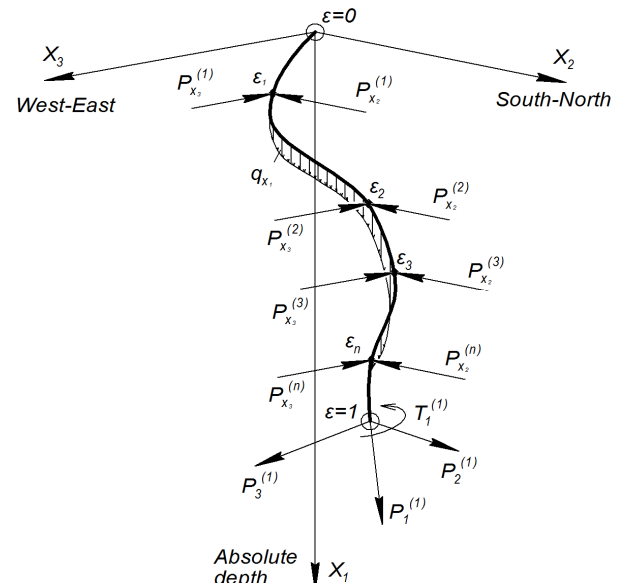
$$T = \mu + \sum_{v=1}^p T^{(v)} \delta(\varepsilon - \varepsilon_v),$$

where  $q$  is the vector of distribute force;  $P^{(i)}$  is the vector of force;  $\mu$  is the vector of a distributed moment;  $T^{(v)}$  is the vector of a moment;  $\varepsilon_i, \varepsilon_v$  are curvilinear coordinates of points where vectors are affixed.

For the practical implementation the system (1) – (5) can be projected on the axes of the moving coordinate system [9]. For this purpose we examine the components of external forces and moments vectors in details.

Among the distributed forces which influence a flexible beam we examine the equivalent distributed force of a drill string weight in a drilling agent  $q_{x1}$  (see Fig. 4). This force is always directed vertically downward parallel to the axes  $OX_1$ . That is why its projections on the moving coordinate system axes equal

$$q_1 = q_{x1} l_{11}, \quad q_2 = q_{x1} l_{21}, \quad q_3 = q_{x1} l_{31}.$$



**Figure 4 – Simulation of a drill string by a flexible beam**

We simulate reactions between sides of a well and a drill string in the form of the centered forces  $P_{x2}^{(i)}$  and  $P_{x3}^{(i)}$ . These forces are parallel to axes  $OX_2$  and  $OX_3$  of a fixed coordinate system (see Fig. 4). Their projections on moving coordinate system axes are the following

$$P_1^{(i)} = l_{12} \sum_{i=1}^n P_{x2}^{(i)} + l_{13} \sum_{i=1}^n P_{x3}^{(i)},$$

$$P_2^{(i)} = l_{22} \sum_{i=1}^n P_{x2}^{(i)} + l_{23} \sum_{i=1}^n P_{x3}^{(i)},$$

$$P_3^{(i)} = l_{32} \sum_{i=1}^n P_{x2}^{(i)} + l_{33} \sum_{i=1}^n P_{x3}^{(i)}.$$

The values  $l_{12}$ ,  $l_{13}$ ,  $l_{22}$ ,  $l_{23}$ ,  $l_{32}$  and  $l_{33}$  are components of the matrix  $L$ .

Well side reactions  $P_2^{(1)}$ ,  $P_3^{(1)}$  and the axial force  $P_1^{(1)}$  in point  $\varepsilon = 1$  (see Fig. 4) are simulated by the moving forces, which are in boundary conditions according to recommendations [9].

We assume that distributed moments do not influence a drill string. Therefore,

$$\mu = 0.$$

A reactive torsional moment, which occurs in a drill string, is simulated by the torsional moment  $T_1^{(1)}$ , which influences a flexible beam bottom (see Fig. 4). This moment is also in boundary conditions [9].

We assume that the top ( $\varepsilon = 0$ ) ending of a flexible beam is fixed and the bottom one ( $\varepsilon = 1$ ) is free. Therefore, the boundary conditions for solving the system of vector equations of equilibrium (1) – (5) are the following

$$\begin{aligned} u_1(0) &= 0, \quad u_2(0) = 0, \quad u_3(0) = 0, \\ \vartheta_1(0) &= \gamma_0, \quad \vartheta_2(0) = 0, \quad \vartheta_3(0) = 0, \\ Q_1(1) &= P_1^{(1)}, \quad Q_2(1) = P_2^{(1)}, \quad Q_3(1) = P_3^{(1)}, \\ M_1(1) &= T_1^{(1)}, \quad M_2(1) = 0, \quad M_3(1) = 0, \end{aligned}$$

were  $u_i(\varepsilon)$ ,  $Q_i(\varepsilon)$ ,  $M_i(\varepsilon)$  are the projections of a displacement vector, cross-axis force and a bending moment on axes of a moving coordinate system;  $\gamma_0$  is the initial value of a rotary angle;  $i = 1, 2, 3$ .

In order to compare an elastic line of a flexible beam with a well axis we define the following projections of a displacement vector on a fixed coordinate system

$$\begin{aligned} u_{x_1}(\varepsilon) &= l_{11}u_1(\varepsilon) + l_{21}u_2(\varepsilon) + l_{31}u_3(\varepsilon), \\ u_{x_2}(\varepsilon) &= l_{12}u_1(\varepsilon) + l_{22}u_2(\varepsilon) + l_{32}u_3(\varepsilon), \\ u_{x_3}(\varepsilon) &= l_{13}u_1(\varepsilon) + l_{23}u_2(\varepsilon) + l_{33}u_3(\varepsilon). \end{aligned}$$

The sum of concentrated forces  $P_{x_2}^{(1)}$ ,  $P_{x_2}^{(2)}$ ,  $P_{x_2}^{(3)}$ , ...,  $P_{x_2}^{(n)}$ ,  $P_{x_3}^{(1)}$ ,  $P_{x_3}^{(2)}$ ,  $P_{x_3}^{(3)}$ , ...,  $P_{x_3}^{(n)}$ , their values and points of application are defined by the following algorithm. Using the results of a drillhole survey there is chosen a potentially dangerous area of a well. Usually this is an interval with the greater change of inclination and horizontal angles. Points at the beginning and the ending of a "dog-leg" are fixed. Then the middle point of a "dog-leg" is put. Then the top of a flexible beam is fixed at the point directly over the middle point. This point is called a top point. The length of a drill string is chosen in the way that its bottom reaches the point, which is under the middle point. This point is called a bottom point. This length is equal to the distance between the top and the bottom points along the well axis.

Consequently, the sequence of calculation is the following. We assume that the distances between curvilinear coordinates  $\varepsilon_1, \varepsilon_2, \varepsilon_3, \dots, \varepsilon_n$  along the well axis are known and constant.

Two forces  $P_{x_2}^{(1)}$  and  $P_{x_3}^{(1)}$  are affixed to point  $\varepsilon_1$ . The values of these forces change in a predetermined range. Now we solve the equations (1) – (5) and define the function projections of an elastic axis of a flexible beam for each plane  $X_1OX_2$ ,  $X_1OX_3$  and  $X_2OX_3$ . If three

projections exist simultaneously, each point of which is under the conditions from Table 1, we calculate the potential energy of bending by the formula

$$U = \sum_{i=2}^3 \int_0^{l_b} \frac{M_i(l_b)}{2EI} dl, \quad (6)$$

where  $EI$  is the stiffness of the drill string bending.

Then two forces  $P_{x_2}^{(2)}$  та  $P_{x_3}^{(2)}$  are applied to the point  $\varepsilon_2$ . The calculation, given above, is repeated. A similar algorithm is implemented for all curvilinear coordinates.

The next step is to load the flexible beam simultaneously with two pairs of forces. To begin with, the first pair of forces  $P_{x_2}^{(1)}$  and  $P_{x_3}^{(1)}$  is applied to the point  $\varepsilon_1$ , the second – to the point  $\varepsilon_2$ . By varying the values of these forces in a given range we obtain solutions of vector differential equations of flexible beam equilibrium. As it was mentioned before, the potential energy of the beam deformation is calculated for solutions that meet the conditions of the Table 1.

Further, the position of the first pair of forces remains unchanged, and the second pair of forces is applied to the point  $\varepsilon_3$ . All the necessary calculations are made, followed by the movement of the second pair of forces to the point with curvilinear coordinates  $\varepsilon_4$ . The calculation is made again. This process is repeated until the second pair of forces is applied to all points except  $\varepsilon_1$ .

Then the first pair of forces is applied in turn to point  $\varepsilon_2$ , and the second pair – to each point except for  $\varepsilon_2$ . All the necessary calculations are made for each combination of reciprocal positions of forces pairs. Then the first pair of forces is concentrated in points  $\varepsilon_3, \varepsilon_4, \dots, \varepsilon_n$  in turn. However, as in the previous case, the calculation is made for all the points of the second pair of forces except for the points, where the first pair exists. In general, the calculation is repeated until the number of forces that are simultaneously applied to a flexible beam is equal to  $n$ . Of all the possible positions of an elastic axis of a flexible beam we choose the one for which the potential energy of bending is minimal.

Then the flexible beam length  $l_b$  increases. For this purpose, its top moves to one drillhole survey point towards the wellhead, and its bottom – toward the bottom hole. The calculation is made according to the given above algorithm. Having made the calculations we obtain the next position of an elastic axis.

The length of a beam increases until the dependence of  $U(l_b)$  acquires an explicit minimum. The function of an elastic axis, which corresponds to this minimum, is the desired position of a drill string in the "dog-leg".

In practice this algorithm is calculated in "Waterloo Maple."

Now we shall consider the way of applying this methodology. According to the drillhole survey of the well № 10 in Odessa oilfield there is found a "dog-leg" with a sharp change in zenith and azimuth angles in curvilinear depth interval of 990–1,065 m.

We analyze the stress-strain state of a drill string with outer diameter of 127 mm and inner diameter of

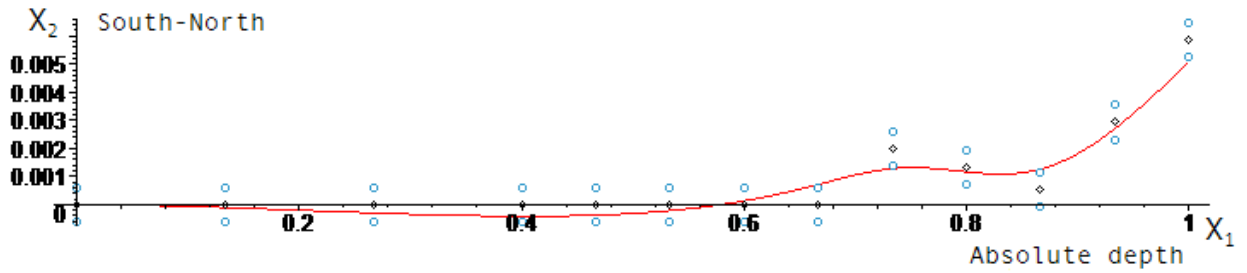


Figure 5 – The projection of an elastic axis of a drill string in a “dog leg” of the well № 10 in Odessa oilfield on the plane  $X_1OX_2$

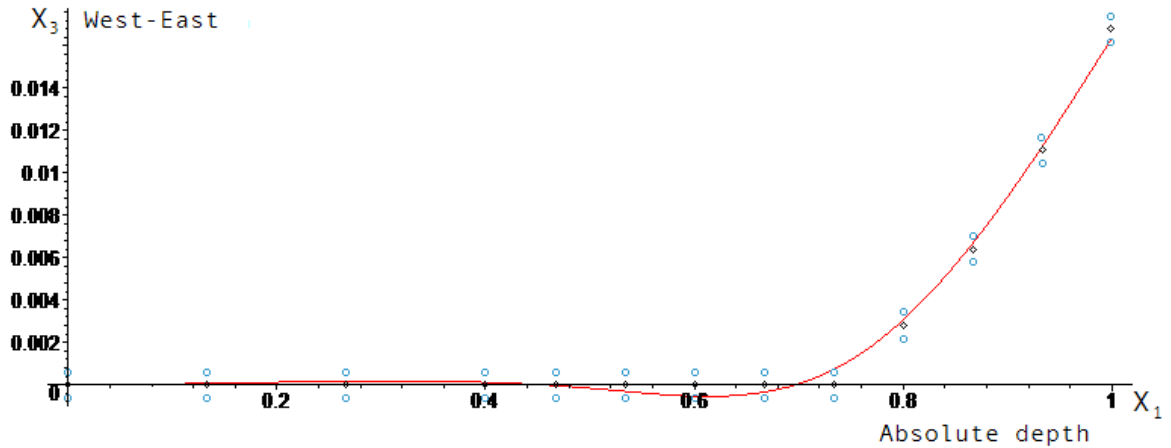


Figure 6 – The projection of an elastic axis of a drill string in a “dog leg” of the well № 10 in Odessa oilfield on the plane  $X_1OX_3$

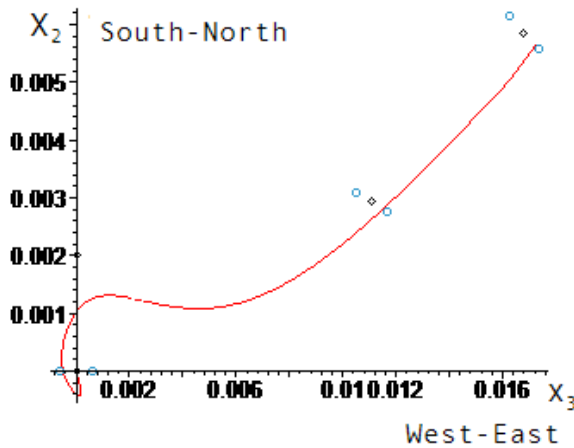


Figure 7 – The projection of an elastic axis of a drill string in a “dog leg” of the well № 10 in Odessa oilfield on the plane  $X_2OX_3$

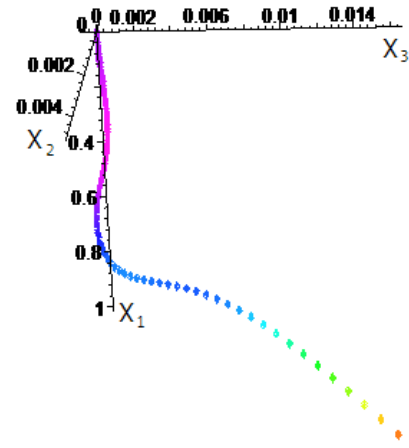


Figure 8 – An elastic axis of a drill string in a 3D coordinate system

111 mm in this area. The diameter of the hole in a “dog-leg” is 219 mm.

Using the above algorithm we obtain the function of an elastic axis of a drill string. Its projections on the planes  $X_1OX_2$ ,  $X_1OX_3$  and  $X_2OX_3$  are shown in Figures 5, 6 and 7 respectively.

The view of an elastic axis of a drill string in a 3D coordinate system is shown in Figure 8.

The number, values and curvilinear coordinates of forces, which show the reactions of borehole walls, are shown in Table 2.

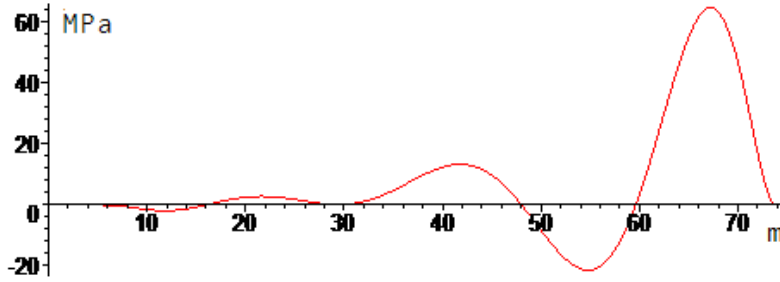
The length of a flexible beam that satisfies the criterion of the minimum potential energy of bending (6) equals  $l_b = 75$  m

The diagram of the change of normal stresses of bending in the cross section of a drill string is shown in Figures 9 and 10.

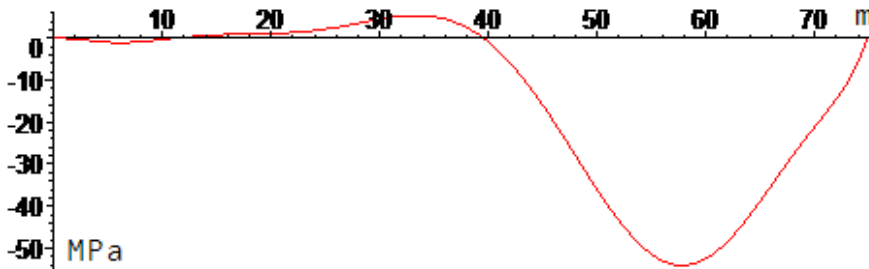
So we can see that the “dog-leg” in this well causes bending strains around principal axes of inertia of a drill string cross-section that are equal to 65 MPa and 54 MPa.

**Table 2 – Values and curvilinear coordinates of forces that show reactions of borehole walls**

| Forces parallel to axis $x_2$                          |      |        |       |      |        |     |
|--|------|--------|-------|------|--------|-----|
| Forces values, N                                       | 0    | -1,660 | 3,310 | 0    | -2,450 | 700 |
| Points curvilinear coordinates, where forces are apply | 0.50 | 0.67   | 0.74  | 0.75 | 0.86   | 1   |
| Forces parallel to axis $x_3$                          |      |        |       |      |        |     |
| Forces values, N                                       | 370  | 0      | 0     | -770 | 0      | 410 |
| Points curvilinear coordinates, where forces are apply | 0.50 | 0.67   | 0.74  | 0.75 | 0.86   | 1   |



**Figure 9 – The diagram of the change of normal stresses of bending around axis  $e_3$**



**Figure 10 – The diagram of the change of normal stresses of bending around axis  $e_2$**

We also define bending strains in this section of the well without taking into account a “dog-leg”. The results of a drillhole survey are the following: the length of a well curvilinear section  $l_c = 180$  m; the zenith and azimuth angles at the beginning of the interval  $\alpha_s = 0^\circ$ ,  $\gamma_s = 24^\circ$ ; the zenith and azimuth angles at the end of interval  $\alpha_e = 65^\circ$ ,  $\gamma_e = 120^\circ$ . For calculating the strains we use the formulas mentioned in the research paper [10]. Therefore, the change of a spatial angle is

$$\Delta Q = \frac{180^\circ}{\pi} (\cos \alpha_s \cos \alpha_e + \sin \alpha_s \sin \alpha_e \sin(\gamma_e - \gamma_s)) = 24^\circ .$$

Radius of the well curvature equals

$$R = \frac{57.3 l_c}{\Delta Q} = 429.8 \text{ m} .$$

Therefore bending strains of a drill string cross-section are the following

$$\sigma_b = \frac{E d}{2 R} = 0.31 \cdot 10^8 \text{ Pa} . \quad (7)$$

Thus, a “dog-leg” of a hole axis causes the fact that the strain in the cross section of a drill string is twice greater than the value, calculated by the formula (7). It should also be noted that, based on the system (1) – (5), the offered method has a number of features, including:

1) curvilinear coordinates are taken as an argument of an elastic axis of a drill string. It enables us to analyze the stress-strain state when the movement of an elastic axis relatively the initial position is commensurate with the length of the studied column. This may be appropriate when analyzing “dog-legs” in the curvilinear sections of wells;

2) the directions of vectors of some external loads are chosen depending on the specifics of their actions in real conditions. For example, a vector of gravity of a drill string is set by one projection in a *fixed* coordinate system and is directed vertically downwards along the axis  $X_1$ . Therefore, when analyzing the stress-strain state of the column, the values of the vector’s projections on the *moving coordinate system* can vary depending on the position of an elastic axis. This fact enables us to consider the influence of gravity on deformation of drill pipes in both the axial and radial directions. The vectors of external axial force and torsional moment, applied to the lower end of the drill string, however, are set in projections of a *moving coordinate system*. This makes it possible to preserve their directions relatively to the cross-section of drill pipes regardless the deformation of the elastic axis of the column;

3) a view of a function of the elastic axis is not set first, it is the result of the numerical solution of differential equations of equilibrium.

From the above mentioned we can conclude on the feasibility of additional research of stress-strain state of the drill string in the place of a well with a sharp change in inclination and horizontal angles.

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## Напружено-деформований стан бурильної колони на ділянках свердловин з “різкими перегинами”

*Р.В.Рачкевич* \*

*Івано-Франківський національний технічний університет нафти і газу;  
вул. Карпатська, 15, м. Івано-Франківськ, 76019, Україна*

Бурильна колона змодельована гнучким стержнем, що зазнає просторового згину під дією прикладених навантажень. Для аналізу напружено-деформованого стану використано систему векторних диференціальних рівнянь рівноваги. Математична модель враховує власну вагу та жорсткість бурильної колони, осьову силу, крутний момент і реакції стінки свердловини.

На основі розробленого методу проаналізовано напружено-деформований стан бурильної колони в “різкому перегині” свердловини за реальних умов експлуатації. Одержано функцію пружної осі бурильної колони в тривимірній системі координат. Побудовано графіки нормальних напружень згину навколо головних осей інерції поперечного перерізу бурильної колони та відзначено, що наявність “різкого перегину” збільшує величину цих напружень у два рази порівняно із величиною, яка б мала місце за відсутності “різкого перегину” свердловини.

Ключові слова: *бурильна колона, гнучкий стержень, напружено-деформований стан, нормальні напруження, просторова пружна вісь, “різкий перегин”*.