The method of segmentation of stochastic cyclic signals for the problems of their processing and modeling

I. V. Lytvynenko

Ternopil Ivan Pul'uj National Technical University; 56, Ruska str., Ternopil, 46000, Ukraine

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Abstract

The segmentation method (signal partitioning into specific sections, segments) of a cyclic stochastic signal is considered. Taking into account cyclicity attributes, segmental cyclic and segmental zone structure, there is proposed a method for solving the segmentation problem of a cyclic stochastic signal with a stable or replaceable rhythm. The information obtained on the segment structure allows us to analyze the rhythm: to estimate the value of the period in the case of a stable rhythm, and in the case of a variable rhythm - the rhythmic structure (discrete rhythm function). There are given the examples of segmentation of modeled and real cyclic signals, and the accuracy of the developed segmentation method is estimated.

The developed method can be used in digital process automation systems (diagnostics and forecasting) of cyclic data: cardiac signals of various physical nature, cyclic economic processes, gas consumption and energy consumption processes, surface processes of relief formations of modern materials.

Keywords: *cyclic random process, rhythmic structure, segmentation, segmental zone structure, segmental cyclic structure..*

Introduction

In many branches of science and technology, there are various vibrational phenomena and cyclic signals, determining the relevance of their research, analysis, forecasting and modeling. For example, heart work is characterized by the cyclic deployment of the phase of cardiac muscle contractions in the time course, the respiratory process is characterized by rhythmic, cyclic oxygen saturation, the economic cyclical processes and processes of energy consumption, for example, electric, water, gas, oil consumption are characterized by cyclic deployment and repeatability over time.

Recorded cyclic signals contain important information that is reflected in the corresponding segments of implementation that characterize the stages, the time-based cyclic process. Therefore, there arises the question of its partitioning into certain specific segments (zones, cycles, areas) in many problems of analyzing such signals. In particular, the tasks of segmenting a cyclic signal take place during the morphological analysis of cardiac signals, the analysis of cardiac rhythm by cardiointervalogram, obtained both during the patient's rest and physical activity, such problems arise during the analysis of cyclic economic data and others [1–4, 7] Information about the segmental, zone structure of the continuous cyclic signal allows to select the step of its sampling [5], to evaluate the rhythmic structure (to determine the rhythm function) of the cyclic signal [6], to carry out its statistical processing, analysis and simulation [1, 4, 7]. Therefore, automated detection and analysis of such time segments on a registered cyclic signal is an important and relevant scientific and technical task, the solution of which allows creating new Digital Processing Systems for cyclic data that carry out diagnostics or a prediction.

There are many methods of segmentation [8–13] developed for specific cyclic signals of different physical nature, for example, electrocardiosignal, magnetocardiosignal, reocardial signal, and others. In practice, metric methods of segmentation in the time domain are often used: the method of amplitude characteristics of the signal [8], the method of form function [10], the method of standards [11], and others. One of the most common methods for separating QRS segments (QRS complexes) is the method of analyzing the difference function of the first order (the first derivative) and the difference function of the second order, and comparing their extrema with threshold values [9]. Although there are many developed methods of segmentation, in practice, they do not always correctly process real cyclic signals because these methods do not have adaptation mechanisms for the features of the cyclic signal, which are necessary, for example, in pathologies, where the splitting of the key segments (R-zones) of the electrocardiosignal occurs or pathologies, associated with changes in rhythm. Also, there is a problem of the impossibility of using known methods in the tasks of coherent processing cyclic signals of various physical nature, due to different methods, which are based on different mathematical models and therefore there is no a single approach and

* Corresponding author: iaroslav.lytvynenko@gmail.com

methodology of segmentation of various cyclic signals. This necessitates the creation of new methods for processing cyclic signals, in particular, the methods of segmentation, from the standpoint of a single theoretical and methodological approach that will eliminate existing problems.

This work is devoted to the development of a method for segmentation of a stochastic cyclic signal, the model of which is a cyclic random process with a segmental structure.

Mathematical model of stochastic cyclic signals

In [14] there is given the definition of a cyclic random process of a continuous argument and there is formulated the problem of segmentation.

Definition 1 (a cyclic random process). A separable random process $\xi(\omega,t),\ \omega\in\mathbf{\Omega},\ t\in\mathbf{R}$ is called a cyclic random process of a continuous argument if there exists a function $T(t,n)$ satisfying conditions (1) and (2) of the rhythm function that the finite-dimensional vectors $(\xi(\omega,t_1),\xi(\omega,t_2),\ \ldots\ ,\ \xi(\omega,t_k))$ and $(,\xi(\omega,t_1+T(t_1,n)),\quad \xi(\omega,t_2+T(t_2,n))),\ \ldots\ ,\ \xi(\omega,t_k+T(t_k,n))),n\in\mathbf{Z}$, where $\{t_1,t_2,\ldots,t_k\}$ is the set of separability of the process $\xi(\omega,t),\ \omega\in\mathbf{\Omega},\ t\in\mathbf{R}$, for all the whole $k\in\mathbf{N}$ are stochastically equivalent in the broad sense.

The function of rhythm $T(t,n)$ determines the law of changing the time intervals between single-phase values in different cycles of the cyclic process.

The function $T(t,n)$ must satisfy the following properties:

a) $T(t,n)>0$, if $n>0$ ($T(t,1)<\infty$);

b) $T(t,n)=0$, if $n=0$; (1)

c) $T(t,n)<0$, if $n<0$, $t\in\mathbf{R}$.

For any $t_1\in\mathbf{W}$ and $t_2\in\mathbf{W}$, for which $t_1<t_2$, for the function $T(t,n)$ there is the following strict inequality:

$$T(t_1,n)+t_1<T(t_2,n)+t_2,\forall n\in\mathbf{Z}\ . \qquad (2)$$

Here is a definition of a cyclic random process with a segmental structure.

Definition 2 (a random process with the segmental (zone) structure). Let us have a vector N of arbitrary random processes, given on the same probabilistic space

$$\Xi_{\xi}(\omega,t)=\left\{\xi_i(\omega,t),\ i=\overline{1,N},\ \omega\in\mathbf{\Omega},\ t\in\mathbf{R}\right\}, \qquad (3)$$

and non-random splitting $\mathbf{D_R}=\left\{\mathbf{W}_i, i=\overline{1,N}\right\}$ of the domain of definition $\mathbf{R}$, which is connected with the indicator functions $\left\{I_{\mathbf{W}_i}(t),\ i=\overline{1,N}\right\}$, signified according to the following expression

$$I_{\mathbf{W}_i}(t)=\begin{cases}1, & t\in\mathbf{W}_i\\ 0, & t\notin\mathbf{W}_i,\ i=\overline{1,N},\end{cases} \qquad (4)$$

And the half-intervals $\mathbf{W}_i=[t_i,t_{i+1})$ satisfy the following conditions:

$$\bigcup_{i=1}^{N}\mathbf{W}_i=\mathbf{R},\ \mathbf{W}_i\neq\varnothing,\ \mathbf{W}_i\bigcap\mathbf{W}_j=\varnothing, \qquad (5)$$
$$\forall i\neq j,\ i,j=\overline{1,N}$$

Then the random process presented by such a construction

$$\xi(\omega,t)=\sum_{i=1}^{N}\hat{\xi}_i(\omega,t)I_{\mathbf{W}_i}(t),\ \omega\in\mathbf{\Omega},\ t\in\mathbf{R}, \qquad (6)$$

is called a random process with a segmental zone structure (from N segments-zones).

Note that the random process (6) with the segmental zone structure can be given in the following form:

$$\xi(\omega,t)=\sum_{i=1}^{N}\bar{\xi}_i(\omega,t),\omega\in\mathbf{\Omega},t\in\mathbf{R}, \qquad (7)$$

where $\left\{\bar{\xi}_i(\omega,t),\ i=\overline{1,N},\ \omega\in\mathbf{\Omega},\ t\in\mathbf{R}\right\}$ is the set of random processes, which are defined by

$$\bar{\xi}_i(\omega,t)=\xi(\omega,t)I_{\mathbf{W}_i}(t),\ \omega\in\mathbf{\Omega},\ t\in\mathbf{R}. \qquad (8)$$

The components $\left\{\bar{\xi}_i(\omega,t),\ i=\overline{1,N},\ \omega\in\mathbf{\Omega},\ t\in\mathbf{R}\right\}$ of the construction (7) and the components $\left\{\hat{\xi}_i(\omega,t),\ i=\overline{1,N},\ \omega\in\mathbf{\Omega},\ t\in\mathbf{R}\right\}$ of the construction (6) are interconnected by the following relations:

$$\bar{\xi}_i(\omega,t)=\begin{cases}\hat{\xi}_i(\omega,t), t\in\mathbf{W}_i,\\ 0, t\notin\mathbf{W}_i.\end{cases} \qquad (9)$$
$$i=\overline{1,N},\ \omega\in\mathbf{\Omega},\ t\in\mathbf{R}$$

or they are

$$\bar{\xi}_i(\omega,t)=\hat{\xi}_i(\omega,t)I_{\mathbf{W}_i}(t), \qquad (10)$$
$$i=\overline{1,N},\ \omega\in\mathbf{\Omega},\ t\in\mathbf{R}$$

That is, the corresponding i-th components of the corresponding i-th sets $\mathbf{R}\setminus\mathbf{W}_i$ are equal to zero in the construction (7).

Definition 3. A cyclic random process with a segmental structure is called a process $\xi(\omega,t)\in\mathbf{R},t\in\mathbf{W}$ with a function of rhythm $T(t,n)$, which is presented in the following form (through process cycles, segments-cycles of the process):

$$\xi(\omega,t)=\sum_{i\in\mathbf{Z}}\xi_i(\omega,t),t\in\mathbf{W},\omega\in\mathbf{\Omega}\ . \qquad (11)$$

In practice, by conducting the processing of a cyclic signal it is necessary to determine the whole number of cycles of the investigated signal, that is

$i=\overline{1,C}$, where C is the number of segments-cycles of the cyclic process; $\xi_i(\omega,t)$ corresponds to the i-th cycle of a cyclic random process with a segmental structure, which is defined as follows:

$$\xi_i(\omega,t)=\xi(\omega,t)\cdot I_{\mathbf{W}_i}(t),\quad i=\overline{1,C}, t\in\mathbf{W},\omega\in\mathbf{\Omega} \tag{12}$$

where $I_{\mathbf{W}_i}(t)$ is the indicator function of the i-th cycle, which equals

$$I_{\mathbf{W}_i}(t)=\begin{cases}1, t\in\mathbf{W}_i,\\0, t\notin\mathbf{W}_i.\end{cases} \tag{13}$$

Indicator functions $\left\{I_{\mathbf{W}_i}(t), i=\overline{1,C}\right\}$ are cyclic deterministic numerical functions with functions of the rhythm $T(t,n)$ for a cyclic random process $\xi(\omega,t)$, $\omega\in\mathbf{\Omega}, t\in\mathbf{R}$ with a changeable rhythm in the construction (12), namely:

$$I_{\mathbf{W}_i}(t)=I_{\mathbf{W}_i}(t+T(t,n)), i=\overline{1,C}, t\in\mathbf{W}, n\in\mathbf{Z}. \tag{14}$$

In case, if we assume that the indicator functions $\left\{I_{\mathbf{W}_i}(t), i=\overline{1,C}\right\}$ are periodic deterministic functions with period in the construction (12) T:

$$I_{\mathbf{W}_i}(t)=I_{\mathbf{W}_i}(t+T), i=\overline{1,N}, t\in\mathbf{R}, \tag{15}$$

then the cyclic random process $\xi(\omega,t)$, $\omega\in\mathbf{\Omega}, t\in\mathbf{R}$ has a a constant rhythm.

The sets $\mathbf{W}_i$ for determining the indicator function of the i-th cycle of the process are determined through the half-interval for the case of a continuous process, that is $\mathbf{W}=\mathbf{R}$:

$$\mathbf{W}_i=[t_i,t_{i+1}), \tag{16}$$

where t_i is the time instant of the beginning of the i-th cycle of the process.

For the case of a discrete process $\mathbf{W}=\mathbf{D}$ the sets $\mathbf{W}_i$ for determining the indicator function of the i-th cycle of the process are determined as follows:

$$\mathbf{W}_i=\left\{t_{i,l}, l=\overline{1,L}\right\}, i=\overline{1,C}, \tag{17}$$

where L is the number of discrete starting points per cycle, $L=const$.

A cyclic random process can also be presented in this form (through the zones of the process, segments-zones of the process):

$$\xi(\omega,t)=\sum_{i\in\mathbf{Z}}\sum_{j=1}^{Z}\xi_{i_j}(\omega,t), t\in\mathbf{W},\ \omega\in\mathbf{\Omega}, \tag{18}$$

where Z is the number of segments-zones in each cycle of the cyclic process; $\xi_{i_j}(t), t\in\mathbf{W}_{i_j}$ is the j-th zone in the i-th cycle of the cyclic random process, equal to:

$$\xi_{i_j}(\omega,t)=\xi(\omega,t)\cdot I_{\mathbf{W}_{i_j}}(t)=\xi_i(\omega,t)\cdot I_{\mathbf{W}_{i_j}}(t),\quad i=\overline{1,C}, j=\overline{1,Z}, t\in\mathbf{W} \tag{19}$$

where $I_{\mathbf{W}_{i_j}}(t)$ is the indicator function of the j-th zone in the i-th cycle, equal to:

$$I_{\mathbf{W}_{i_j}}(t)=\begin{cases}1, t\in\mathbf{W}_{i_j},\\0, t\notin\mathbf{W}_{i_j}.\end{cases} \tag{20}$$

The sets $\mathbf{W}_{i_j}$ for determining the indicator function of the j-th zone in the i-th cycle of the process are determined through the half-intervals for the case of a continuous process, that is, $\mathbf{W}=\mathbf{R}$:

$$\mathbf{W}_{i_j}=\left[t_{i_j},t_{i_{j+1}}\right), \tag{21}$$

where t_{i_j} is the time instant of the beginning of the j-th zone in the i-th cycle of the process.

For the case of a discrete process $\mathbf{W}=\mathbf{D}$, the domains $\mathbf{W}_i$ of the indicator function of the i-th cycle of the process are determined as follows:

$$\mathbf{W}_{i_j}=\{t_{i_j,l}, l=\overline{1,L_j}\}, L=\sum_{j=1}^{Z}L_j,\quad i=\overline{1,C}, j=\overline{1,Z} \tag{22}$$

where L is the number of discrete starting points per cycle, $L=const$; L_j is the number of discrete starting points on the j - th zone.

The process (11) corresponding to the i-th cycle of the cyclic random process is associated with the process (19), which corresponds to the j-th zones of the cyclic random process, by the following dependence:

$$\xi_i(\omega,t)=\sum_{j=1}^{Z}\xi_{i_j}(\omega,t),\quad t\in\mathbf{W},\omega\in\mathbf{\Omega},\forall i=\overline{1,C} \tag{23}$$

The domains for determining the segments-zones and segments-cycles of the process with a segmental structure satisfy the following relation:

$$\mathbf{W}_i=\bigcup_{j=1}^{Z}\mathbf{W}_{i_j}, \bigcup_{i=1}^{C}\bigcup_{j=1}^{Z}\mathbf{W}_{i_j}=\mathbf{W}, \mathbf{W}_{i_j}\neq\varnothing,\quad \mathbf{W}_{i_{j_1}}\bigcap\mathbf{W}_{i_{j_2}}=\varnothing, j_1\neq j_2. \tag{24}$$

Segmental structure of cyclic signals

The segmental structure of cyclic signals is divided into a segmental zone (zone-cyclic structure) $\mathbf{D}_z$ and a segmental cyclic structure $\mathbf{D}_c$, while the segmental zone structure is embedded into the segmental cyclic structure. In practice, there are analyzed the following cyclic signals in cyclic data processing systems: cardiac signals, cyclic signals of relief formations, economic cycles, solar activity cycles, etc., the mathematical models of which are random processes of one fixed signal implementation. Such implementations can be considered as some of the deterministic cyclic functions. Therefore, we will continue to consider the generalized segmental structure as a deterministic segmental structure of cyclic signals, that is, the time moments,

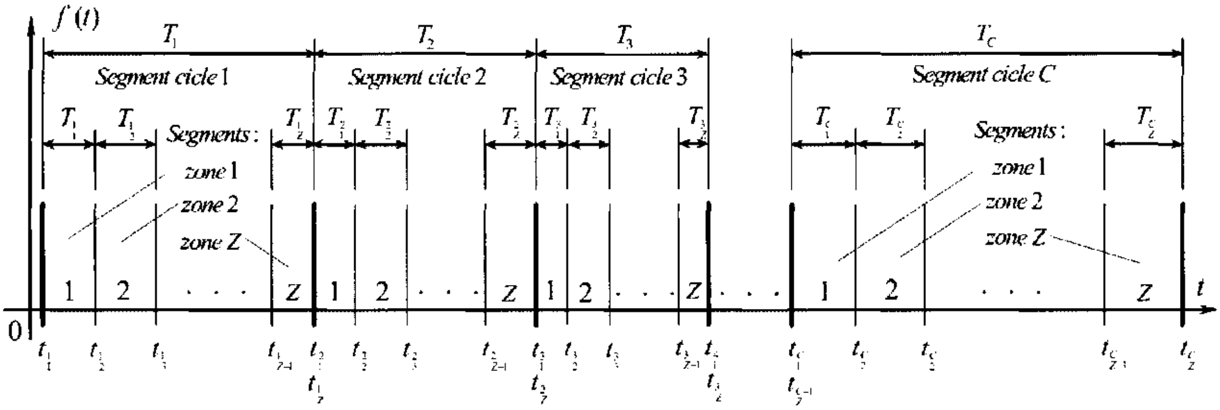


Figure 1 – Schematic representation of the segment structure for the implementation of a cyclic signal

corresponding to the boundaries of the segments-cycles or segments-zones, are not random variables.

The zone-cyclic structure of the cyclic stochastic signal is given by a plurality of time points corresponding to the beginning of the segments-zones of the cyclic signal (segmental zone structure):

$$\mathbf{D}_z = \left\{ t_{i_j}, i = \overline{1,C}, j = \overline{1,Z} \right\}, \; t_i = t_{i_1}, \forall i = \overline{1,Z}. \quad (25)$$

When it is possible to allocate only the set of time starting points, corresponding to the beginning of the segments of the cycles in the investigated signal, then the segment structure is given through its cycles (a segmental cyclic structure):

$$\mathbf{D}_c = \left\{ t_i, i = \overline{1,C} \right\}. \quad (26)$$

Let's consider the work of the zone-cyclic structure as it includes the segmental cyclic structure. Figure 1 shows a schematic representation of the segmental zone structure of a cyclic signal with the lengths of segments-cycles $\left\{ T_i, i = \overline{1,C} \right\}$, the length of segments-zones $\left\{ T_{i_j}, i = \overline{1,C}, j = \overline{1,Z} \right\}$, and the boundaries of the beginning of the j-th segment t_{i_j} and its end $t_{i_{j+1}}$ in each cycle.

Estimation of the rhythmic structure of cyclic signals

Based on the information about the segmental structure of the cyclic signal, we can estimate its rhythmic structure. The rhythmic structure of the cyclic signal is represented by a discrete function of the rhythm [6, 15]. The discrete function of the rhythm is embedded in the continuous function of the rhythm, the starting points of which correspond to the starting points of a specific segmental structure, and the values of the discrete function of the rhythm are determined as follows (for the case when the segment structure reflects the zone-cyclic structure and is determined by the moments of the beginning of the segments-zones of the signal):

$$T(t_{i_j}, n) = t_{i_{j+n}} - t_{i_j}, \forall i = \overline{1,C}, j = \overline{1,Z}, n \in \mathbf{Z}, \quad (27)$$

where n is the number of cycles through which one-phase values of the investigated signal are remote. For our case, we take $n = 1$, that is, single-phase samples of the first, second, third, etc. cycles of the signal studied.

The discrete function of the rhythm of a cyclic signal, the segment structure of which reflects the cyclic structure, will be determined by the moments of the start of signal:

$$T(t_i, n) = t_{i+n} - t_i, \forall i = \overline{1,C}, n \in \mathbf{Z}. \quad (28)$$

Thus, the information about the beginning of the segments-zones and segments-cycles of a cyclic signal allows to determine its discrete rhythmic structure, the information of which is contained in the discrete function of rhythm $T(t_{i_j}, n)$, which is embedded in the continuous function of the rhythm $T(t,n)$ of a cyclic signal of a continuous argument. Figure 2 shows a schematic representation of the rhythmic structure (a discrete function of the rhythm), and a continuous piecewise linear function of the rhythm (dotted line).

The statement of the problem of segmenting stochastic cyclic signals is given in [14]. It includes the concept of the attribute of a cyclic signal. An attribute or attributes for a cyclic signal is a characteristic that indicates a stochastic equivalence between single-phase cycle values [15].

The problem of segmentation of stochastic cyclic signals

The essence of the problem of cyclic signals segmentation consists in determining the partition of the set of a cyclic signal. That is, in finding an unknown set of temporal (spatial) starting points of j-th segments-zones in the corresponding i-th segments-cycles $\mathbf{D}_z = \left\{ t_{i_j}, i = \overline{1,C}, j = \overline{1,Z} \right\}$, or a set of temporal (spatial) starting points $\mathbf{D}_c = \left\{ t_i, i = \overline{1,C} \right\}$ – the corresponding i-th segments-cycles, that is, in finding the partition of the set $\mathbf{W}_{\mathbf{W}}^{z} = \left\{ \mathbf{W}_{i_j}, i = \overline{1,C}, j = \overline{1,Z} \right\}$ of the cyclic signal or $\mathbf{W}_{\mathbf{W}}^{c} = \left\{ \mathbf{W}_i, i = \overline{1,C} \right\}$.

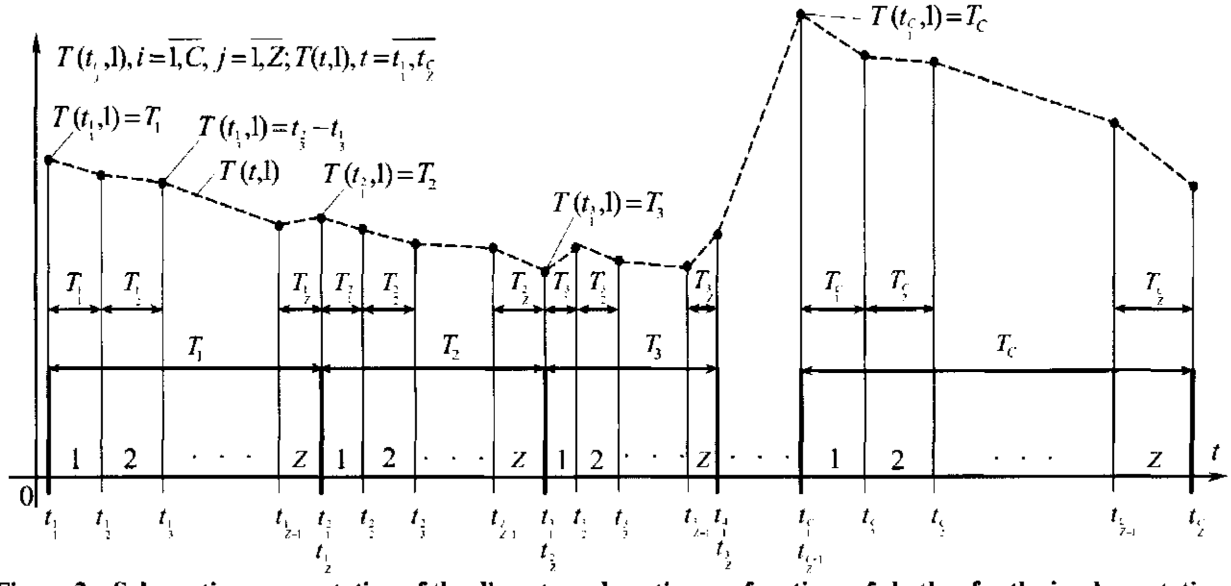


Figure 2 – **Schematic representation of the discrete and continuous functions of rhythm for the implementation of a cyclic signal (a continuous function of rhythm, piecewise linear, marked with a dotted line)**

In this case, it is necessary that there were fulfilled the conditions for the bijection of the cycle starting points and the strict improvement of the starting points (29) that correspond to the segments, as well as the equality of the segments starting points attributes (30) for a given set of time points $\mathbf{D}_z = \left\{t_{i_j}, i=\overline{1,C}, j=\overline{1,Z}\right\}$ or $\mathbf{D}_c = \left\{t_i, i=\overline{1,C}\right\}$ that is, the segment zone structure is the following for the set (25):

$$t_{i_j} \leftrightarrow t_{i_{j+1}},\ldots;\ t_{i_{j+1}} > t_{i_j},\ t\in \mathbf{W}, i=\overline{1,C}, j=\overline{1,Z}, \quad (29)$$

$$p(f(t_{i_j})) = p(f(t_{i_{j+1}})) \to \mathbf{A},\ t\in \mathbf{W}, i=\overline{1,C}, j=\overline{1,Z}. \quad (30)$$

The segmental cyclic structure is the following for the set (26):

$$t_i \leftrightarrow t_{i+1},\ldots;\ t_{i+1} > t_i,\ t\in \mathbf{W}, i=\overline{1,C}, \quad (31)$$

$$p(f(t_i)) = p(f(t_{i+1})) \to \mathbf{A},\ t\in \mathbf{W}, i=\overline{1,C}, \quad (32)$$

where $\mathbf{A}$ is a set of attributes [15].

Method of segmentation of stochastic cyclic signals

Consider the developed method of segmentation step by step. To ensure the same results of segmentation, we assume that the test signal has undergone a preliminary processing in which the trend component was eliminated and the beginning of the signal is synchronized with the beginning of the reference frame (the beginning of the cycle count).

Estimation of segment structure (preliminary segmentation) for input data $\xi_\omega(t_k), t_k \in \mathbf{W} = \mathbf{D}$ – discrete realization of a cyclic stochastic signal,

$$S(t_m) = \frac{1}{m}\sum_{k=1}^{m}\xi_\omega(t_k) - \frac{1}{K-m}\sum_{k=m+1}^{K}\xi_\omega(t_k),$$
$$m=\overline{1,K-1},\ k=\overline{1,K},\ t_k\in \mathbf{W}, \quad (33)$$

where $S(t_m)$ is the value of statistics in the discrete m moment of time; K is the number of registered implementations; $\xi_\omega(t_k)$ is the value of a sample implementation at a k moment of time.

After calculating the statistics, the search for its local extremises is carried out by one of the known methods. A set of time points is obtained that corresponds to extremes of statistics:

$$\mathbf{S}_u = \left\{\tilde{t}_u, u=\overline{1,U}\right\}, \quad (34)$$

A set of time points, corresponding to the maxima of statistics:

$$\mathbf{S}_{\max} = \left\{\tilde{t}_q, q=\overline{1,Q}\right\}. \quad (35)$$

A set of time points, corresponding to the minima of statistics:

$$\mathbf{S}_{\min} = \left\{\tilde{t}_r, r=\overline{1,R}\right\}. \quad (36)$$

For the sets (35) and (36), the condition holds:

$$\mathbf{S}_u = \mathbf{S}_{\max}\bigcup \mathbf{S}_{\min}, U=Q+R, \quad (37)$$

The applied statistics (33) responds to sharp changes in mathematical expectation and has proved itself well at certain moments of the time of stochastic signal disruption, the model of which is a piecewise-stationary random process. After receiving the sets (35) and (36), we determine which of them will be used for further segmentation steps.

At the stage of the previous segmentation, we select the first count according to the following conditions:

$$\text{If } \tilde{t}_q < \tilde{t}_r,\ q=1, r=1, \text{ then } \tilde{t}_1 = t_k, k=1; \quad (38)$$

$$\text{then } \tilde{t}_s = \tilde{t}_q, s=2,\ldots, q=1,\ldots, \text{ we use } \mathbf{S}_{\max}. \quad (39)$$

$$\text{If } \tilde{t}_q > \tilde{t}_r,\ q=1, r=1, \text{ then } \tilde{t}_1 = t_k, k=1; \quad (40)$$

$$\text{then } \tilde{t}_s = \tilde{t}_r, s=2,\ldots, r=1,\ldots, \text{ we use } \mathbf{S}_{\min}. \quad (41)$$

After this procedure we get a preliminary breakdown of the cyclic stochastic signal where the initial data are $\tilde{\mathbf{D}}_s = \left\{\tilde{t}_s, s=\overline{1,S}\right\}$ – set of counts of time

Table 1 – Usage of attributes for specifying the cyclic structure

Attributes	Analytical record
Values are in the range of values	$p(\xi_\omega(t))=\xi_\omega(\hat{t}_1)-\Delta\le\xi_\omega(t)\le\xi_\omega(\hat{t}_1)+\Delta$, the range of possible values 2Δ

Table 2 – Specifying the starting points of cycles' segments by using the matrics of close values of the starting points of these segments

Matrics of close values (absolute)	Matrics of close values (Euclidean)
$\tilde{t}_k$ – a starting point being specified, t_e – a starting point for specifying, $\min\rho=\lvert\xi_\omega(\tilde{t}_k)-\xi_\omega(t_e)\rvert$. Terms of reference for specifying: If a starting point being specified is in the middle of the signal $t_e=\begin{cases}t_{k-e}, k\ge 1+e;\\ t_{k+e}, k\le K-e.\end{cases}$ If a starting point being specified is at the end or at the beginning of the signal: $t_e=\begin{cases}t_{k-e}, k=K;\\ t_{k+e}, k=1.\end{cases}$	$\tilde{t}_k$ – a starting point being specified, t_e – a starting point for specifying, $\min\rho=\sqrt{(\xi_\omega(\tilde{t}_k)-\xi_\omega(t_e))^2}$. Terms of reference for specifying: If a starting point being specified is in the middle of the signal $t_e=\begin{cases}t_{k-e}, k\ge 1+e;\\ t_{k+e}, k\le K-e.\end{cases}$ If a starting point being specified is at the end or at the beginning of the signal: $t_e=\begin{cases}t_{k-e}, k=K;\\ t_{k+e}, k=1.\end{cases}$

points of segments (segments of cycles and segments of zones). The next stage is cyclic structure evaluation.

Cyclic structure evaluation (segment cyclic srtucture) with input data $\tilde{\mathbf{D}}_s=\{\tilde{t}_s, s=\overline{1,S}\}$ – a set of time segments. Since a starting point of the reference signal is synchronized with a starting point of the first cycle, we accept $\tilde{t}_1$ as a starting point of the investigated signal, $\hat{t}_1=\tilde{t}_i=\tilde{t}_s, i,s=1$ starting point of the beginning of the first cycle.

In the next step, we evaluate a starting point of cycles by attribute according to Table 1. Since it is known that the values of cyclic signals under investagation are of the normal distribution law [15], then the range of search for single-phase values will be set as $D=3\sqrt{d}$ where d is the value of the variance for the investigated signal, which is being determined and affects the accuracy of segmentation.

At this stage, it is necessary to fulfil the conditions of equality for attributes and to fulfil the isomorphism of cycles starting points, that is,

$$\tilde{t}_g=\begin{cases}\tilde{t}_k, \text{ if } p(\xi_\omega(\hat{t}_1))=p(\xi_\omega(\tilde{t}_k))\to A,\\ g=2\ldots, k=\overline{2,K},\\ \tilde{t}_k - \text{counting is not taken into account,}\end{cases} \quad (42)$$

$$\tilde{t}_i=\begin{cases}\tilde{t}_g, \text{ if }\begin{cases}p(\xi_\omega(\hat{t}_1+l))=p(\xi_\omega(\tilde{t}_g+l)),\\ g=\overline{2,G-1},\ \tilde{t}_g\le l\le\tilde{t}_{g-1},\\ \hat{t}_1+l<\tilde{t}_g+l,\ g=\overline{2,G-1},\\ l-\text{counts within the segment-cycle,}\end{cases}\\ \tilde{t}_g - \text{counting is not taken into account,}\end{cases} \quad (43)$$

The initial data of this stage is a set of time starting points that correspond to the beginnings of cycles.

Clarification of reference segments of cycles with input data $\tilde{\mathbf{D}}_c=\{\tilde{t}_i, i=\overline{1,\tilde{C}}\}$ and starting points, being specified $\tilde{t}_k, \tilde{t}_i$. When specifying the readings, it is necessary to take into account the range of specifying $\tilde{t}_i<t_e<\tilde{t}_{i+1}$, where t_e is a starting point to specify and e the number of starting points for specifying.

The initial data of this stage are: $\tilde{\mathbf{D}}_c=\{\tilde{t}_i, i=\overline{1,\tilde{C}}\}$ – a set of time starting points that correspond to the specified start of cycles; $\tilde{C}$ – the number of estimated cycles at this stage, while the segments of the zone on the cycles are not specified.

Specifying and formation of the segmental structure (band-cyclic structure or segmental cyclic structure) with input data $\tilde{\mathbf{D}}_c=\{\tilde{t}_i, i=\overline{1,\tilde{C}}\}$ and $\tilde{\mathbf{D}}_s=\{\tilde{t}_s, s=\overline{1,S}\}$. This stage is divided into two sub-stages. A set of time starting points $\tilde{\mathbf{D}}_z=\{\tilde{t}_{ij}, i=\overline{1,C}, j\in\mathbf{Z}\}$ is being formed that are between time intervals between the cycles (this set will be specified).

1. Specifying the segmental structure (segmental zone structure). If there are no reference segments of the loops between the set limits of the segments of the loops, that is, the condition of equality according to the attribute is fulfilled $p(\xi_\omega(\tilde{t}_1))=p(\xi_\omega(\tilde{t}_2))=\ldots=$ $=p(\xi_\omega(\tilde{t}_i))=\ldots=p(\xi_\omega(\tilde{t}_{\tilde{C}})), i=\overline{1,\tilde{C}}$, then the test

signal contains only the reference segments of the cycles and does not contain counts of smaller segment segments (within the cycles) then $\hat{\mathbf{D}}_c = \tilde{\mathbf{D}}_c = \{\hat{t}_i = \tilde{t}_i, i = \overline{1, C = \tilde{C}}\}$, therefore $\hat{\mathbf{D}}_c = \left\{\hat{t}_i, i = \overline{1, C}\right\}$, we obtain the segmental cyclic structure and pass to const 3.4.

Otherwise, if there are one or more reference segments-zones between the cycles of segment segments, for example, a condition is fulfilled, $p(\xi_\omega(\tilde{t}_1)) = p(\xi_\omega(\tilde{t}_3)) = p(\xi_\omega(\tilde{t}_5)) = ... = p(\xi_\omega(\tilde{t}_i)) = = ... = p(\xi_\omega(\tilde{t}_{\tilde{C}})), i = \overline{1, \tilde{C}}$ – starting points of cycles segments and correspondingly $p(\xi_\omega(\tilde{t}_2)), p(\xi_\omega(\tilde{t}_4))$ – segments of cycle zones, thus we get zone cyclic structure (segment zone structure).

We conduct the evaluation of the received reference points for equality on the attribute, and fulfill the conditions for the isomorphism of reference zones.

Let's take $\tilde{t}_{i_j} = \tilde{t}_i, i, j = 1$ – as a starting point of the first zone in the first cycle.

$$\tilde{t}_{i_g} = \begin{cases} \tilde{t}_k, \text{if } p(\xi_\omega(\hat{t}_1)) = p(\xi_\omega(\tilde{t}_k)) \to \mathbf{A}, \\ g = 2..., k = \overline{2, K}, j = 2... \\ \tilde{t}_k - \text{counting is not taken into account,} \end{cases} \quad (44)$$

$$\tilde{t}_{i_j} = \begin{cases} \tilde{t}_g, \text{if} \begin{cases} p(\xi_\omega(\hat{t}_1 + l)) = p(\xi_\omega(\tilde{t}_g + l)), \\ g = \overline{2, G-1}, \tilde{t}_g \le l \le \tilde{t}_{g+1}, \\ \hat{t}_1 + l < \tilde{t}_g + l, g = \overline{2, G-1}, \\ l - \text{counts within the segment-cycle,} \end{cases} \\ \tilde{t}_g - \text{counting is not taken into account.} \end{cases} \quad (45)$$

In the segment structure, it is stated that the number of readings of the segments of the zones on each cycle is the same, so the number of zones in the cycles should also be the same. Given this for a set $\tilde{\mathbf{D}}_z = \{\tilde{t}_{i_j}, i = \overline{1, C}, j = \overline{1, \tilde{Z} + 1}\}$, this set does not take into account starting point cycles. Where $\tilde{Z}$ is the number of areas of zones to be specified.

The following is justified:

if the number of specified reference zones for each cycle is the same, then the number of zones in the cycles is the same and is equal to $Z_i = Z$ then $\tilde{\mathbf{D}}_z = \{\tilde{t}_{i_j}, i = \overline{1, C}, j = Z_i = Z\}$; where Z is the number of zones for which there was equality by the attribute between the starting points of the cycles boundaries; Z_i is the number of zones in the i cycle.

We combine the set of cycles starting points $\hat{\mathbf{D}}_c = \left\{\hat{t}_i, i = \overline{1, C}\right\}$ and the received set of starting points of zones-segments $\tilde{\mathbf{D}}_z = \{\tilde{t}_{i_j}, i = \overline{1, C}, j = Z_i = Z\}$, thus we get a set of starting points $\hat{\mathbf{D}}_z = \{\hat{t}_{i_j}, i = \overline{1, C}, j = \overline{1, Z}\}$, which takes into account both starting points of the cycles and the starting points of the zones

$$\hat{\mathbf{D}}_c \bigcup \tilde{\mathbf{D}}_z = \hat{\mathbf{D}}_z. \quad (46)$$

We go to paragraph 3.

If the number of zone starting points in each cycle is not the same (this may be due to the choice of the sampling rate of the investigated signal), as a consequence of not equalizing the values of the attribute, then it is necessary to specify the zone starting points of paragraph 2.

2. Specifying of starting points of segments of zones with input data $\tilde{\mathbf{D}}_z = \{\tilde{t}_{i_j}, i = \overline{1, C}, j = \overline{1, \tilde{Z}}\}$ and starting points being specified $\tilde{t}_k$, $\tilde{t}_{i_j}$. To specify starting points it is necessary to set a range of specifying, equal to $2e$, where e is a number of starting points for specifying.

After specifying zones' starting points we combine sets of cycles' starting points $\hat{\mathbf{D}}_c = \left\{\hat{t}_i, i = \overline{1, C}\right\}$ and obtained specified zones' starting points $\tilde{\mathbf{D}}_z = \{\tilde{t}_{i_j}, i = \overline{1, C}, j = Z_i = Z\}$ we get $\hat{\mathbf{D}}_z = \{\hat{t}_{i_j}, i = \overline{1, C}, j = \overline{1, Z}\}$ – which takes into account both starting points of the cycles and the starting points of the zones.

The input data at this stage are as follows $\hat{\mathbf{D}}_z = \{\hat{t}_{i_j}, i = \overline{1, C}, j = \overline{1, Z}\}$, where Z is a number of evaluated zones or $\hat{\mathbf{D}}_c = \left\{\hat{t}_i, i = \overline{1, C}\right\}$, where C is the number of evaluated cycles.

3. Analysis of the rhythm, which can be stable or changeable in cyclic signals. For example, when we deal with a stable rhythm, the values of time (spatial) distances, the intervals between single-phase values are constant values, and the duration of the cycles can be considered as the duration of the period, as in the case of periodic deterministic signals (cyclic signals). In the case where the distances between the single-phase values are different in different cycles, that is, they are different in terms of the length of the cycles, then we are dealing with a signal characterized by a changeable rhythm.

The input data for this stage is a segmental zone structure $\hat{\mathbf{D}}_z = \{\hat{t}_{i_j}, i = \overline{1, C}, j = \overline{1, Z}\}$ or a segmental cyclic structure $\hat{\mathbf{D}}_c = \left\{\hat{t}_i, i = \overline{1, C}\right\}$.

Having received the segmental structure, we deal with the rhythm analysis, which can be done either according to Part 3.1 or 3.2.

3.1. If $t_{i+1_j} - t_{i_j} = t_{i+1} - t_i = \text{const} = T - time$, $\forall i = \overline{1, C}, j = \overline{1, Z}$ is a constant rhythm, i.e. $\xi_\omega(t) = \xi_\omega(t + nT)$.

We evaluate the value of the period by the following formula:

$$\hat{T} = t_{i+1_j} - t_{i_j} = t_{i+1} - t_i, \forall i = \overline{1, C}, j = \overline{1, Z} \quad (47)$$

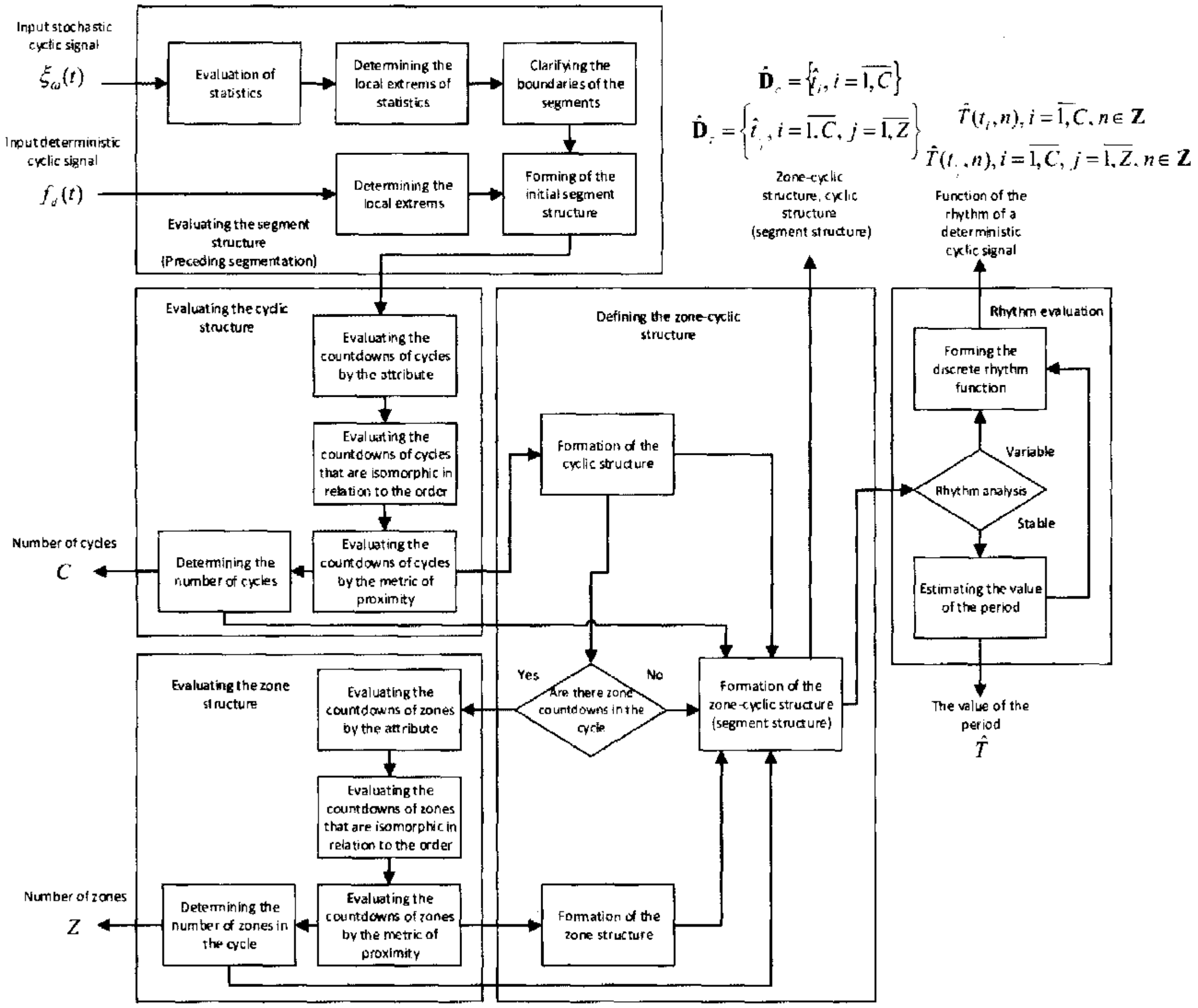


Figure 3 – Algorithmic support for the method of segmentation of stochastic cyclic signals

You can submit a discrete function of rhythm, which is defined through the evaluated period:

- for the starting points of segments-zones,

$$\hat{T}(t_{ij}, n) = n \cdot \hat{T}, \forall t_{ij} \in \mathbf{W}, i = \overline{1,C}, j = \overline{1,Z}, n \in \mathbf{Z}; \quad (48)$$

- for the starting points of segments-cycles,

$$\hat{T}(t_i, n) = n \cdot \hat{T}, \forall t_i \in \mathbf{W}, i = \overline{1,C}, n \in \mathbf{Z}. \quad (49)$$

3.2. If $t_{i+1j} - t_{ij} \neq t_{i+1} - t_i \neq \text{const}$, $\forall i = \overline{1,C}, j = \overline{1,Z}$ is a changeable rhythm, that is $\xi_\omega(t) \neq \xi_\omega(t + nT)$, and so $\xi_\omega(t) = \xi_\omega(t + T(t,n))$.

We evaluate the discrete function of rhythm by formulas (27) and (28).

Taking into account the above described algorithm, Figure 3 presents a structural diagram of the algorithmic support of the developed method of segmentation. The developed method is implemented in the form of software in Delphi (programming language).

The developed method allows us to get information about the segment structure: $\hat{\mathbf{D}}_z = \{\hat{t}_{ij}, i = \overline{1,C}, j = \overline{1,Z}\}$ or $\hat{\mathbf{D}}_c = \{\hat{t}_i, i = \overline{1,C}\}$, to evaluate the rhythm (rhythmic structure) in the case of a stable rhythm, to estimate the value of the period; and in the case of a changeable one - to evaluate the discrete function of the rhythm.

Results of application of the developed method

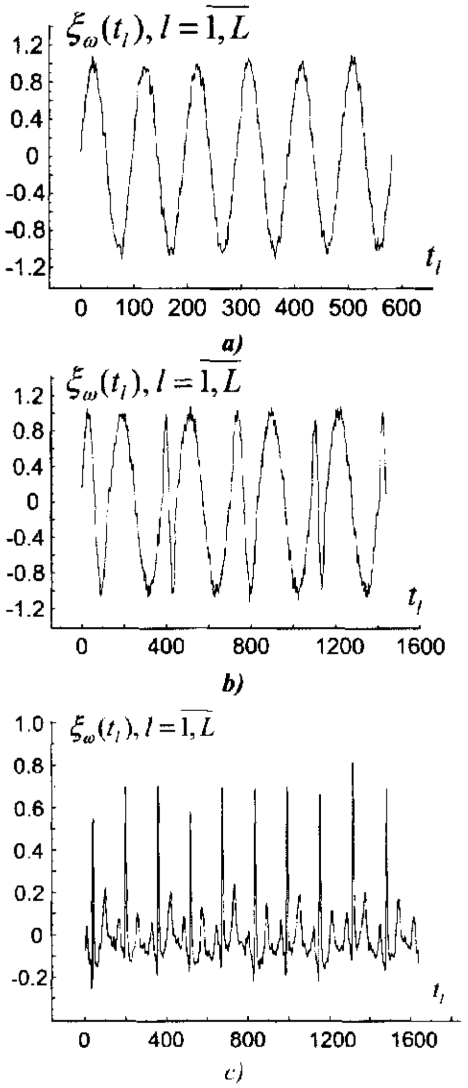
Figure 4 shows, as an example, the test signals of stochastic cyclic signals, in conditional units. For checking the developed segmentation method, they were modeled taking into account the functions of rhythm, shown in Figure 5.

Applying the developed method of segmentation to the test signals we received a segmental structure and evaluated the functions of the rhythm.

The absolute and relative errors of segmentation were determined by the formulas:

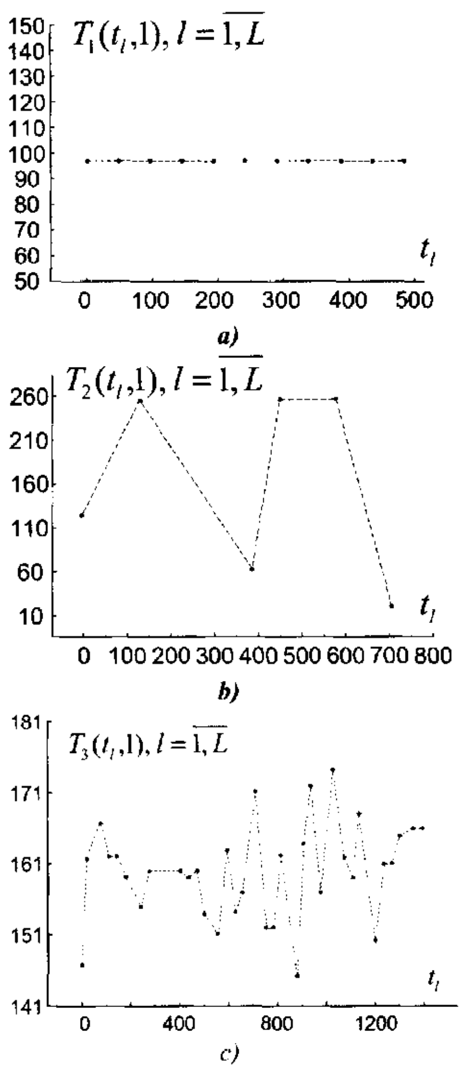
$$\Delta(t_k) = \sqrt{\frac{1}{L}\sum_{l=1}^{L}(t_l - \hat{t}_l)^2}, \ k = \overline{1,L}, \quad (50)$$

$$\delta(t_k) = \frac{\Delta(t_k)}{\sqrt{\frac{1}{L}\sum_{l=1}^{L}\hat{t}_l^2}} \cdot 100\%, \ k = \overline{1,L}, \quad (51)$$



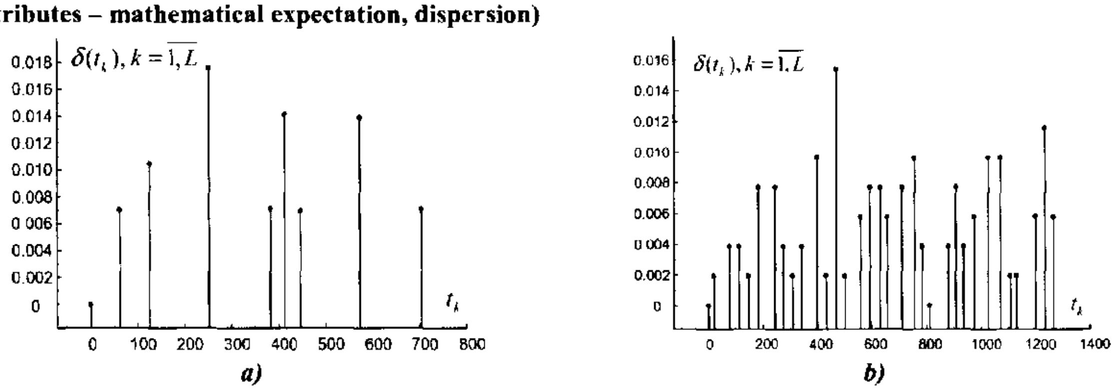
a) a stochastic cyclic signal (a stable rhythm);
b) a stochastic cyclic signal (changeable rhythm);
c) realization of the electrocardiosignal, diagnosis - conditionally healthy person

Figure 4 – Simulated test cyclic signals – for assessing the accuracy of the method of segmentation of stochastic cyclic signals (attributes – mathematical expectation, dispersion)



a) a stochastic cyclic signal (a stable rhythm, period ($T = 97$); b) a stochastic cyclic signal (a changeable rhythm); c) realization of the electrocardiosignal, diagnosis - a conditionally healthy person, a dotted line – a constant rhythm function

Figure 5 – Discrete rhythm functions of simulated test cyclic signals



a) a stochastic cyclic signal (a changeable rhythm); b) implementation of an electrocardiosignal, diagnosis – a conditionally healthy person

Figure 6 – Root-mean-square errors of the determined starting points of the discrete rhythm function for the cyclic signals

Table 4 – Errors of the defined starting points of the discrete rhythm functions of the cyclic signals.

Segmentation of stochastic cyclic signals	The absolute error (not more than)	The relative error (not more than), %
A stochastic cyclic signal (a stable rhythm)	0	0
A stochastic cyclic signal (a changeable rhythm)	8.2	1.8
A stochastic cyclic signal (a changeable rhythm), implementation of an electrocardiosignal	11.4	2.1

where t_l is a starting point of the simulated discrete function of the rhythm (used to simulate the cyclic signal); $\hat{t}_l$ is a starting point of the moment of time of a definite discrete rhythm function by the method of segmentation; L is the number of starting points of the discrete function of the rhythm, $l=\overline{1,L}$; t_k is a starting point of absolute and relative errors, $k=\overline{1,L}$.

Since a partial case can be a cyclic signal, modeled by a stochastically periodic process, this method can be used both for segmentation and estimation of the period value. However, it is sometimes sufficient to determine the period by one of the known methods and we get the segmental structure for cyclic periodic signals.

Conclusions

In this paper, the method of segmentation of a stochastic cyclic signal is developed on the basis of information about its segmental structure and attributes. The developed method allows to segment cyclic signals, the mathematical model of which is a cyclic random process with a segmental structure.

This method of segmentation complements the apparatus for processing cyclic signals and can be used in real cyclic data automated processing digital systems in medicine, economics and other industries for the tasks of segmentation. The method allows us to evaluate a stable or a changeable changeablerhythm: in the case of a stable rhythm, to evaluate the period's value, and in the case of a changeable rhythm, to evaluate the rhythmic structure. The estimation of the accuracy of stochastic cyclic signals segmentation method is given; the relative error of segmentation for the investigated signals does not exceed 2.1%.

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Метод сегментації стохастичних циклічних сигналів для задач їх обробки та моделювання

Л.В. Литвиненко

Тернопільський національний технічний університет імені Івана Пулюя; вул. Руська, 56, м. Тернопіль, 46000, Україна

Розглядається метод сегментації (розбиття сигналу на характерні ділянки, сегменти) циклічного стохастичного сигналу. Враховуючи атрибути циклічності, сегментну циклічну та сегментну зонну структури, запропоновано метод вирішення проблеми сегментації циклічного стохастичного сигналу з стабільним чи змінними ритмом. Отримані відомості про сегментну структуру дозволяють проаналізувати ритм: у випадку стабільного ритму оцінити значення періоду, а у випадку змінного ритму – ритмічну структуру (дискретну функцію ритму). Наведено приклади сегментації змодельованих та реальних циклічних сигналів, дана оцінка точності розробленого методу сегментації.

Розроблений метод може бути використаний в автоматизованих системах цифрової обробки (діагностики та прогнозу) циклічних даних: кардіосигналів різної фізичної природи, циклічних економічних процесів, процесів газоспоживання та енергоспоживання, процесів поверхні рельєфних утворень сучасних матеріалів.

Ключові слова: *ритмічна структура, сегментація, сегментна зонна структура, сегментна циклічна структура, циклічний випадковий процес.*