SUM OF ENTIRE FUNCTIONS OF BOUNDED
L-INDEX IN DIRECTION

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We use definitions and denotations from [1] and [2].

It is known that a product of two entire functions of bounded $L$-index in
direction is a function with the same class (see [2], [4]). But the class of entire
functions of bounded index is not closed under addition. The example was
constructed by W. Pugh (see [3] and also [5]). Recently we generalized Pugh’s
example for entire functions of bounded $L$-index in direction [4].

Meanwhile, there are sufficient conditions of index boundedness for a sum of
two entire functions [3].

In this report we present sufficient conditions of boundedness of $L$-index in
direction for a sum of entire functions. They are new for entire functions of
bounded $l$-index too.

We consider an arbitrary hyperplane $A = \{ z \in \mathbb{C}^n : \langle z, c \rangle = 1 \}$, where
$\langle c, b \rangle \neq 0$. Obviously that $\bigcup_{z_0 \in A} \{ z_0 + t b : t \in \mathbb{C} \} = \mathbb{C}^n$.

Let $z_0 \in A$ be a given point. If $F(z_0 + t b) \neq 0$ as a function of variable $t \in \mathbb{C}$ then there exists $t_0 \in \mathbb{C}$ $F(z_0 + t_0 b) \neq 0$. Thus, for every line $\{ z_0 + t b : F(z_0 + t b) \neq 0 \}$ we fixed one point $t_0$ with specified property. By $B$ we denote
a union of those points $z_0 + t_0 b$ i. e. $B = \bigcup_{F(z^0 + tb) \neq 0} \{ z_0 + t_0 b \}$. Clearly that
for every $z \in \mathbb{C}^n$ there exist $z_0 \in A$ and $t \in \mathbb{C}$ with property $z = z_0 + t b$.

Thus, the next theorem is true.

Theorem 1. Let $L \in Q_b^n$, $\alpha \in (0, 1)$ and $F$, $G$ be the entire in $\mathbb{C}^n$ functions
satisfying conditions

1) $G(z)$ has bounded $L$-index in the direction $b \in \mathbb{C}^n \setminus \{0\}$.
2) for every $z = z_0 + t b \in \mathbb{C}^n$, where $z_0 \in A$, $z_0 + t_0 b \in B$ and $r = |t - t_0| L(z_0 + t b)$ the following inequality is valid

$$\max \left\{ \frac{1}{k! L^k (z_0 + t b)} \left| \frac{\partial^k G(z_0 + t b)}{\partial b^k} \right| : 0 \leq k \leq N_b(G_{\alpha}, L_{\alpha}) \right\} \leq$$

$$\leq \max \left\{ \frac{2r}{L(z_0 + t b)} \right\} \leq$$

$$\leq \max \left\{ \frac{1}{k! L^k (z_0 + t b)} \left| \frac{\partial^k G(z_0 + t b)}{\partial b^k} \right| : 0 \leq k \leq N_b(G_{\alpha}, L_{\alpha}) \right\} .$$
\[ c = \max_{z^0+t_0 \in B} \frac{\|F(z^0+t'b)\| : \|t'-t_0\| = \frac{2\lambda_b(1)}{L(z^0+t_0b)}}{|F(z^0+t_0b)|} < \infty. \]

If \( |\varepsilon| \leq \frac{1-\alpha}{2c} \) then the function

\[ H(z) = G(z) + \varepsilon F(z) \]

is of bounded \( L \)-index in the direction \( b \) with \( N_b(H,L) \leq N_b(G_\alpha,L_\alpha), \) where \( G_\alpha(z) = G(z/\alpha), \) \( L_\alpha(z) = L(z/\alpha). \)

References


[4] Bandura A. I. Product of two entire functions of bounded \( L \)-index in direction is a function with the same class, Bukovyn. Mat. Zh. (in Ukrainian, accepted)


ON FOURIER QUASICRYSTALS

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A Fourier quasicrystal is a pure point complex measure \( \mu \) on \( \mathbb{R}^p \) such that its Fourier transform in the sense of distributions \( \hat{\mu} \) is also a pure point measure. For example, the sum \( \sigma \) of unit masses at the points of \( \mathbb{Z}^p \subset \mathbb{R}^p \) is a Fourier quasicrystal, because \( \hat{\sigma} = \sigma \) in view of the Poisson summation formula. The support of \( \hat{\mu} \) is called spectrum of the Fourier quasicrystal. A set \( S \subset \mathbb{R}^p \) is called uniformly discrete if distances between its distinct points are uniformly bounded away from zero. \( S \) is called a pure crystal, if it is a finite union of translates of a unique full-rank lattice.

At first we show some new conditions for Fourier transform of measures and distributions to be a measure.

Then we consider a Fourier quasicrystal \( \mu \) with discrete support \( \Lambda \). N.Lev, A.Olevskii (2016) proved that if the spectrum of \( \mu \) and the set of differences \( \Lambda - \Lambda \) are both uniformly discrete, then \( \Lambda \) is a subset of a pure crystal.