Definition 1. *Let S_1, S_2 be continuous semilattices with zeros 0_1 and 0_2 resp. An ambiguous representation of S_1 in S_2 is a binary relation $R \subset S_1 \times S_2$ such that*

(a) if $(x, y) \in R$, $x \leqslant x'$ in S_1, and $y' \leqslant y$ in S_2, then $(x', y') \in R$ as well;

(b) for all $x \in S_1$ the set $xR = \{y \in S_2 \mid (x, y) \in R\}$ is non-empty and Scott closed in S_2.

If xRy, then we say that $y \in S_2$ represents $x \in S_1$.

Idea: if $x \in S_1$ holds, then we can (but not obliged) ensure $y \in S_2$.

Hence we construct a category with the continuous semilattices with zeros as the objects and ambiguous representations as arrows. The key problem is to define compositions.

A straightforward attempt to define the composition of $R \subset S_1 \times S_2$, $Q \subset S_2 \times S_3$ as $RQ = \{(x, z) \in S_1 \times S_3 \mid \text{there is } y \in S_2 \text{ such that } (x, y) \in R, (y, z) \in Q\}$, fails because closedness of xRQ in the condition (b) of the definition of ambiguous representation does not always holds.

If we take closures of the values of the corresponding mutivalued mapping $R;Q = \{(x, z) \in S_1 \times S_3 \mid z \in \mathrm{Cl}(xRQ)\}$, then closedness is at hand, but associativity fails!

Based on [4] we shall present a subclass of all ambiguous representations which we called *pseudo-invertible*. Composition of pseudo-invertible representations turned out to be associative, and a new category containing $\mathcal{S}\mathrm{em}_0$ as a subcategory is obtained.

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Entire Dirichlet series with monotonous coefficients and logarithmic h-measure

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Let $\mathcal{L}$ be the class of positive continuous increasing functions on $[0; +\infty)$ and $\mathcal{L}_+$ the subclass of functions $\Phi \in \mathcal{L}$ such that $\Phi(t) \to +\infty$ $(t \to +\infty)$. By φ we denote inverse function to $\Phi \in \mathcal{L}$. By $\mathcal{D}_a$ we denote a subclass of entire Dirichlet series of the form $F(z) = \sum\limits_{n=0}^{+\infty} a_n e^{z\lambda_n}$, $z \in \mathbb{C}$ with a fixed sequence $a = (|a_n|)$, $|a_n| \searrow 0$ $(n_0 \leqslant n \to +\infty)$. Here a sequence (λ_n) such that $\lambda_n \in \mathbb{R}$ $(n \geqslant 0)$, $\lambda_n \neq \lambda_k$ for any $n \neq k$ and $(\forall n \geqslant 0):$ $0 \leqslant \lambda_n < \beta := \sup\{\lambda_j : j \geqslant 0\} \leqslant +\infty$. We denote $\mathcal{D}_a(\Phi)$ subclass of functions $F \in \mathcal{D}_a$ such that $\ln \mu(x, F) \geqslant x\Phi(x)$ $(x \geqslant x_0)$ for $\Phi \in \mathcal{L}$. Let $\mu_n := -\ln|a_n|$ $(n \geqslant 0)$.

Theorem. (O. B. Skaskiv, 1994 [1]). *For every entire function $F \in \mathcal{D}_a$ relation $F(x+iy) = (1+o(1))a_{\nu(x,F)}e^{(x+iy)\lambda_{\nu(x,F)}}$ holds as $x \to +\infty$ outside some set E of finite logarithmic measure, i.e.* log-meas$(E) := \int_E d\ln x < +\infty$, *uniformly in $y \in \mathbb{R}$, if and only if*

$$\sum\nolimits_{n=n_0}^{+\infty} \frac{1}{\mu_{n+1} - \mu_n} < +\infty. \tag{1}$$

It is easy to see that mentioned relation holds for $x \to +\infty$ $(x \notin E)$ uniformly in $y \in \mathbb{R}$, if and only if $M(x,F) \sim \mu(x,F)$ $(x \to +\infty,\ x \notin E)$, hence it follows $M(x,F) \sim m(x,F)$ $(x \to +\infty,\ x \notin E)$.

The finiteness of logarithmic measure of an exceptional set E in previous theorem is the sharp estimate. Then the natural question arises: what conditions must satisfy the entire Dirihlet series $F \in \mathcal{D}_a$ in order to relation $M(x,F) \sim m(x,F)$ holds for $x \to +\infty$ outside some set E of finite logarithmic h-measure, i.e. h-log-meas$(E) < +\infty$?

Theorem 1. *Let (μ_n) be a sequence such that condition* (1) *holds, $h \in \mathcal{L}_+, \Phi \in \mathcal{L}$ and $F \in \mathcal{D}_a(\Phi)$. If*

$$(\forall\ b > 0): \quad \sum\nolimits_{n=n_0}^{+\infty} h\Big(\varphi(\lambda_n) \cdot \Big(1 + \frac{b}{\mu_{n+1} - \mu_n}\Big)\Big) \frac{1}{\mu_{n+1} - \mu_n} < +\infty, \tag{2}$$

then the relation holds as $x \to +\infty$ outside some set E of finite logarithmic h-measure uniformly in $y \in \mathbb{R}$.

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